CBCS SCHEME

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15CS36

Third Semester B.E. Degree Examination, June/July 2018
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks: 80

Note: Answer any FIVE full questions, choosing ONE full question from each module.

Module-1

1 a. Prove that for any propositions p, q, r the compound proposition : $\{p \rightarrow (q \rightarrow r)\} \rightarrow \{(p \rightarrow q) \rightarrow (p \rightarrow r)\}$ is a tautology. **(06 Marks)**

b. Prove the following logical equivalence using the laws of logic: $(p \rightarrow q) \wedge [\neg q \wedge (r \vee \neg q)] \Leftrightarrow \neg (q \vee p)$. **(05 Marks)**

c. Prove the following logical equivalence using the laws of logic: $[\neg p \wedge (\neg q \wedge r)] \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. **(05 Marks)**

OR

2 a. Prove the validity of the arguments using rule of inference.

$$\begin{array}{l} (\neg p \vee \neg q) \rightarrow (r \wedge s) \\ r \rightarrow t \\ \neg t \\ \hline \therefore p \end{array}$$

(05 Marks)

b. Test the validity of the arguments using rule of inference.

$$\begin{array}{l} (\neg p \vee q) \rightarrow r \\ r \rightarrow (s \vee t) \\ \neg s \wedge \neg u \\ \neg u \rightarrow \neg t \\ \hline \therefore p \end{array}$$

(05 Marks)

c. Find whether the following argument is valid:

No Engineering student of 1st or 2nd semester studies logic
Anil is an Engineering student who studies logic

∴ Anil is not in second semester. **(06 Marks)**

Module-2

3 a. Prove by mathematical induction that : $1^2 + 3^2 + 5^2 + \dots + (2n-1)^2 = \frac{1}{3} n (2n-1)(2n+1)$. **(05 Marks)**

b. A sequence $\{C_n\}$ is defined recursively by, $C_n = 3C_{n-1} - 2C_{n-2}$ for all $n \geq 3$ with $C_1 = 5$ and $C_2 = 3$ as the initial conditions, show that $C_n = -2^n + 7$. **(06 Marks)**

c. Determine the coefficient of xyz^2 in the expansion of $(2x - y - z)^4$. **(05 Marks)**

OR

4 a. A certain question paper contains two parts A and B, each containing 4 questions. How many different ways a student can answer 5 questions by selecting atleast 2 questions from each part? **(05 Marks)**

b. Prove by mathematical induction that, for every positive integer n, 5 divides $n^5 - n$. **(06 Marks)**

c. How many numbers greater than 1000000 can be formed by using the digits 1, 2, 2, 2, 4, 4, 0? **(05 Marks)**

Module-3

5 a. Let $f : R \to R$ be defined by

$$f(x) = \begin{cases} 3x - 5 & \text{for } x > 0 \\ -3x + 1 & \text{for } x \le 0 \end{cases}$$

Determine $f^{-1}(0), f^{-1}(1), f^{-1}(-1), f^{-1}(3), f^{-1}(-3), f^{-1}(-6)$, **(06 Marks)**

b. Evaluate S(5, 4). **(05 Marks)**

c. Let f, g, h be the function form R to R defined by $f(x) = x + 2$, $g(x) = x - 2$, $h(x) = 3x$ for all $x \in R$. Find gof, fog, fof, hog, foh. **(05 Marks)**

OR

6 a. Let 'S' be the set of all non-zero integers and $A = S \times S$ on A, define the relation R by (a, b)R(c, d) if and only if ad = bc. Show that 'R' is an equivalence relation. **(06 Marks)**

b. Draw the Hasse diagram representing the positive divisors of 36. **(06 Marks)**

c. Let A = {a, b, c, d, e}. Consider the partition P = {{a, b}, {c, d}, {e}} of A. Find the equivalence relation inducing this partition. **(04 Marks)**

Module-4

7 a. In a survey of 260 college students, the following data were obtained. 64 had taken mathematics course, 94 had taken CS course, 58 had taken EC course, 28 had taken both Mathematics and EC course, 26 had taken both Mathematics and CS course, 22 had taken both CS and EC course, and 14 had taken all three types of course. Determine how many of these students had taken none of the three subjects. **(05 Marks)**

b. Find the rook polynomial for the 3 × 3 board using expansion formula. **(06 Marks)**

c. Solve the recurrence relation :

$a_n + a_{n-1} - 6a_{n-2} = 0$ $n \ge 2$, given $a_0 = -1$ and $a_1 = 8$. **(05 Marks)**

OR

8 a. An apple, a banana, a mango and an orange are to be distributed among 4 boys B_1, B_2, B_3, B_4. The boys B_1 and B_2 do not wish to have an apple, the boy B_3 does not want banana or mango and B_4 refuses orange. In how many ways the distribution can be made so that no boy is displeased. **(06 Marks)**

b. How many permutation of 1, 2, 3, 4, 5, 6, 7, 8 are not derangements? **(04 Marks)**

c. The number of virus affected files in a system is 1000 (to start with) and this increases 250% every two hours. Use a recurrence relation to determine the number of virus affected files in the system after one day. **(06 Marks)**

Module-5

9 a. Define isomorphism. Show that the following graph are isomorphic to each other. Refer Fig.Q9(a). **(06 Marks)**

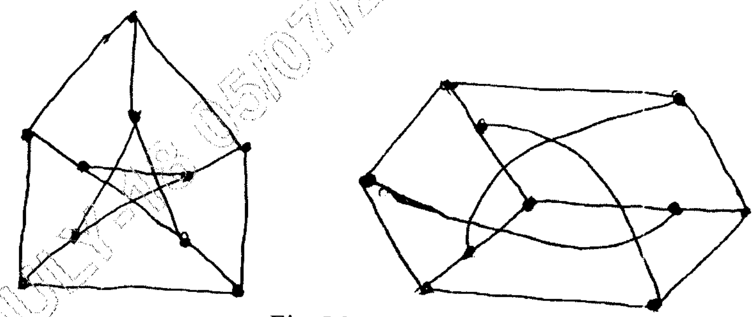

Fig.Q9(a)

b. "A tree with 'n' vertices is having 'n – 1' edges". Prove the given statement. **(05 Marks)**

c. Define complete graph, general graph and Bipartite graph with example for each. **(05 Marks)**

OR

10 a. For a graph with 'n' vertices and 'm' edges, if 'δ' is minimum, 'Δ' is maximum of the degree of vertices. Show that :

$$\delta \le \frac{2m}{n} \le \Delta .$$

(05 Marks)

b. Obtain the optimal prefix code for the message "ROAD IS GOOD". Indicate the code. **(06 Marks)**

c. Apply the merge sort to the following given list of element.
{–1, 0, 2, –2, 3, 6, –3, 5, 1, 4}. **(05 Marks)**

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